Key

**Vector Dynamics#1**

(name)

1. A box, mass 4.0 kg, is pushed along the rough horizontal floor by an applied force of 20.0 N. The coefficient of friction between the floor and the bottom of the box is 0.20.

a) Find the normal reaction force exerted by the floor on the box.

$F_N = F_g = 4\text{kg} \times 9.8\text{N/kg} = 39.2\text{N}$

$F_N$ = 39N ✓

b) Calculate the frictional force acting on the box.

$F_f = \mu F_N$

$= 0.20(39.2\text{N}) = 7.84\text{N}$

$F_f$ = 7.8N ✓

c) Calculate the net horizontal force acting on the box.

$F_{net\,H} = 20.0\text{N} - F_f$

$= 20 - 7.84 = 12.16\text{N}$

$F_{net}$ = 12N ✓

d) Calculate the horizontal acceleration of the box.

$F_{net\,H} = ma$

$12.16\text{N} = 4\text{kg}(a)$, $a = 3.04\text{m/s}^2$

$a_x$ = 3.0m/s² ✓

e) Calculate the time for the box to move 6.0 m from rest.

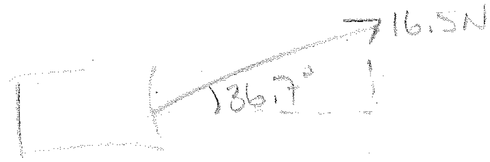
$t = ?$
$d = 6\text{m}$
$a = 3.04\text{m/s}^2$
$V_0 = 0\text{m/s}$

$t = \frac{V_f - V_0}{a} = \frac{6.04\text{m/s}}{3.04\text{m/s}^2}$

$V_f^2 = V_0^2 + 2ad$

$V_f = \sqrt{2(3.04)(6\text{m})} = 6.04$

t = 2.0 s ✓




2. A box, mass 8.44 kg is pulled along a rough horizontal floor by an applied force of 16.5 N acting upwards at 36.7 degrees above the horizontal. The coefficient of friction between the floor and the bottom of the box is 0.149.

a) Calculate the horizontal and vertical components of the applied force.

$F_V = 16.5\,N \sin 36.7° = 9.8608\,N$

$F_H = 16.5\,N \cos 36.7° = 13.229\,N$

horizontal = 13.2 N.
vertical = 9.86 N.

b) Find the normal force exerted by the floor on the box.

$F_N = F_g - F_V$

$= 8.44\,kg(9.8\,N/kg) - 9.86\,N = 72.852\,N$

$F_N$ = 73 N.

c) Calculate the frictional force acting on the box.

$F_f = \mu F_N$

$= 0.149 \times 72.852\,N = 10.8549\,N$

$F_f$ = 10.9 N.

d) Calculate the net horizontal force acting on the box.

$F_{net\,H} = F_H - F_f$

$= 13.229 - 10.8549 = 2.374\,N$

$F_{Net}$ = 2.37 N.

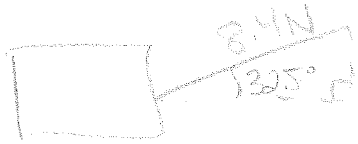
e) Calculate the horizontal acceleration of the box.

$ma = F_{net} = 2.374\,N$

$a = \frac{2.374\,N}{8.44\,kg}$

$a_x$ = 0.28 $m/s^2$.

Answers: 1 a) 39.2 N b) 7.84 N c) 12.6 N d) 3.04 $^m/_s{}^2$ e) ~~1.99~~ 2.0 s 2. a) 13.2 N, 9.86 N b) 73.2 N c) 10.9 N d) ~~2.33~~ 2.37 N e) ~~0.276~~ $^m/_s{}^2$ 0.28 $m/s^2$ f) 6.49 s ?




**Vector Dynamics#2**

Key
(name)

A box, mass 6.54 kg, is pulled along the horizontal floor by an applied force of 8.4 N, which acts upwards at 32.5° to the horizontal. The coefficient of friction between the floor and the bottom of the box is 0.082.

a) Calculate the horizontal and vertical components of the applied force.

$F_H = 8.4N \times \cos 32.5° = 7.084 N$

$F_V = 8.4N \times \sin 32.5° = 4.513 N$

horizontal = 7.1 N ✓
vertical = 4.5 N ✓

b) What is the normal reaction of the floor on the box?

$F_N = F_g - F_V$
$= 6.54 \times 9.8 N - 4.513 N$
$= 59.578685 N$

$F_N$ = 60. N ✓

c) What is the acceleration of the box along the floor?

$F_{net} = ma = F_H - F_f$

$a = \frac{7.084 - 0.082 \times 59.578683 N}{6.54 kg}$

$= 0.3362 m/s^2$

a = 0.34 m/s$^2$ ✓

d) What is the speed of the box after travelling a distance of 6.0 m from rest?

$V_f = ?$
$V_o = 0$
$d = 6m$
$a = 0.34 m/s^2$

$V_f^2 = V_o^2 + 2ad$

$V_f = \sqrt{2(0.34)(6m)}$

$= 2.0 m/s$

v = 2.0 m/s ✓

2. A box, mass 6.54 kg., is pushed along the horizontal floor by an applied force of 8.4 N, which acts downwards at 32.5° to the horizontal. The coefficient of friction between the floor and the bottom of the box is 0.082.

a) Calculate the horizontal and vertical components of the applied force.

$F_V = 8.4\,N \sin 32.5° = 4.513\,N$ [down]

$F_H = 8.4\,N \cos 32.5° = 7.084\,N$

horizontal = 7.1 N.

vertical = 4.5 N. [down]

b) What is the normal force of the floor on the box?

$F_N = F_g + F_V$

$= 6.54 \times 9.8\,N + 4.5\,N$

$= 68.592\,N$

$F_N$ = 69 N.

c) What is the acceleration of the box along the floor?

$F_{net\,H} = F_H - F_f$

$= 7.084\,N - 0.082(68.592\,N)$

$= 1.459\,N$

$a = \frac{F_{net}}{m} = \frac{1.459\,N}{6.54\,kg}$

a = 0.22 m/s$^2$.

d) What is the speed of the box after travelling a distance of 6.0 m from rest?

$V_f = ?$

$V_o = 0$

$d = 6\,m$

$a = 0.22\,m/s^2$

$V_f^2 = 0^2 + 2(0.22\,m/s^2)(6\,m)$

$V_f = 1.625$

v = 1.6 m/s.

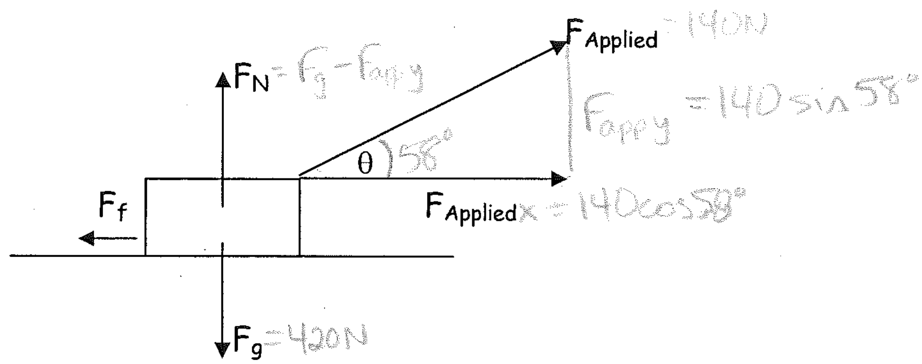
Answers:

1. a) 7.1 N, 4.5 N   b) 60. N   c) 0.34 $^m/_s{}^2$   d) 2.0 $^m/_s$
2. a) 7.08 N, 4.51 N   b) 69 N   c) 0.22 $^m/_s{}^2$   d) 1.6 $^m/_s$

Vector Dynamics#3

Key
(name)

1. A box, weight 420 N, rests on a rough horizontal floor. A force of 140 N is applied to the box horizontally. The box begins to move horizontally when the angle θ is 58° above the horizontal. Determine the coefficient of friction between the floor and the bottom of the box.




$\mu = \frac{F_f}{F_N}$

$= \frac{74.2\,N}{301.27\,N}$

$= 0.25$

$F_{net\,x} = F_{app\,x} - F_f = 0$ ← just starts to move so $F_{app\,x} = F_f$

$0 = 140\cos 58 - F_f$

$F_f = 74.2\ N$

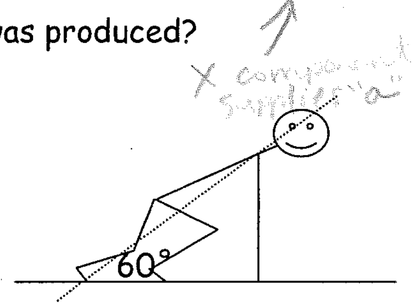
$F_N = F_g - F_{app\,y}$

$= 420 - 140\sin 58$

$= 301.27\ N$

μ = 0.25 ✓

2. A 75 kg track star, at the start of a sprint, pushes on the ground with a measured force of 2000 N at an angle of 60° as shown. What forward acceleration was produced?




$F_{net} =$

$ma = 2000N \cos 60$

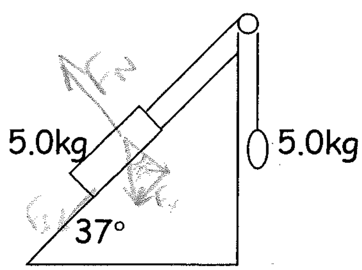
$F_f = \mu \times F_N$ (y-comp) so can ignore

$a = \frac{2000N \cos 60}{75 kg}$

$= 13 \ m/s^2$

a ~~F~~ = 13 m/s² ✓

3. What is the acceleration of the system if the coefficient of friction is 0.15?




$F_{net} = m_T a = 5 \times 9.8N - F_x - F_f$

$a = \frac{5 \times 9.8 - 5 \cdot 9.8 \sin 37° - \mu F_N}{10 Kg}$

$a = \frac{49 - 29.4889 - 0.15(5 \times 9.8 \times \cos 37)}{10}$

$= 1.36 \ m/s^2$

F = 1.4 m/s² ✓

Answer 1. 0.25 2. 13 $^m/_s{}^2$ 3. 1.4 $^m/_s{}^2$

**Vector Dynamics #4** Connected Particles

Key (name)

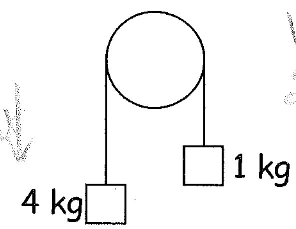
Find the acceleration of each system shown below. Assume no friction, inextensible strings, and massless pulleys. Apply Newton's Second Law ($F_{net} = ma$) to the whole system:

1. What is the total mass being accelerated?

2. What is the net external force acting on the system?

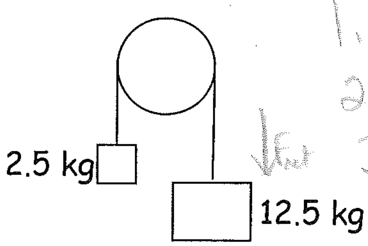
3. Calculate the acceleration.

a)



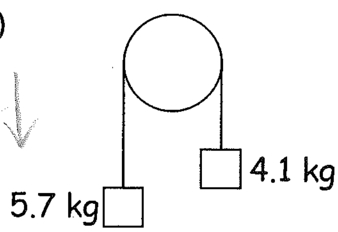

1. 5.0 kg being accelerated
2. $F_{net} = 4kg \times 9.8 - 1kg \times 9.8 = 29.4 N$
3. $F_{net} = ma$ $a = \frac{29.4N}{5kg} = 5.88 m/s^2$

b)



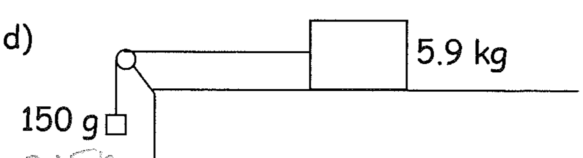

1. 15 kg
2. $F_{net} = 12.5kg \times 9.8 - 2.5kg \times 9.8 = 98N$
3. $a = \frac{98N}{15kg} = 6.5 m/s^2$

c)




1. 9.8 Kg
2. $F_{net} = 5.7 \times 9.8 N - 4.1 \times 9.8 = 15.68 N = 16N$
3. $a = \frac{15.68N}{9.8kg} = 1.6 m/s^2$

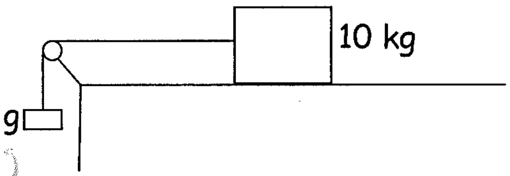
d)




= 0.15 kg

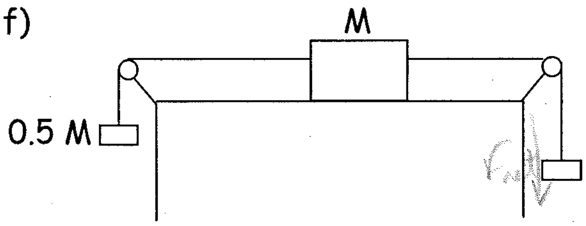
1. 6.05 kg = 6.1 kg
2. $F_{net} = 0.15kg \times 9.8 = 1.47N$
3. $a = \frac{1.47N}{6.05kg} = 0.24 m/s^2$

e)



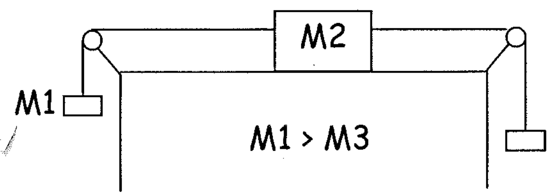

1. $M_{total} = 10.25\text{ kg}$
2. $F_{net} = 0.25\text{ kg} \times 9.8\text{ N/kg} = 2.45\text{ N}$
3. $a = \frac{F_{net}}{m} : \frac{2.45\text{ N}}{10.25\text{ kg}} = 0.24\text{ m/s}^2$

f)



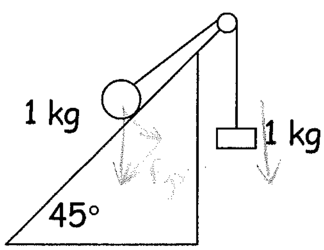

1. $2.25\text{ M}$
2. $F_{net} = 0.75M \times 9.8 - 0.5M \times 9.8 = 2.45\text{ M}$
3. $a = \frac{2.45\text{ M}}{2.25\text{ M}} = 1.1\text{ m/s}^2$

g)




1. $m_1 + m_2 + m_3$
2. $F_{net} = (m_1 - m_3)\,9.8\text{ m/s}^2$
3. $a = \frac{(m_1 - m_3)\,9.8\text{ m/s}^2}{m_1 + m_2 + m_3}$

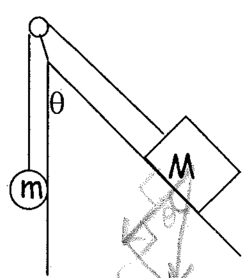
h)




no friction

1. $m_T = 2\text{ kg}$
2. $F_{net} = 1\text{ kg} \times 9.8 - 1\text{ kg} \times 9.8 \sin 45 = 2.87\text{ N}$
3. $a = \frac{2.87\text{ N}}{2\text{ kg}} = 1.44\text{ m/s}^2$

i)




M > m

1. $m + M$
2. $F_{net} = m \times 9.8 - M \sin\theta \times 9.8$
3. $a = \frac{(m - M\sin\theta)\,9.8}{m + M}$

if a + then m > M
if a − then M > m

| Answers: | a | b | c | d | e | f | g | h | i |
|---|---|---|---|---|---|---|---|---|---|
| $M_{total}$(kg) | 5 | 15 | 9.8 | 6.05 | 10.25 | 2.25M | ? | ~~10.25~~ 2kg | M + m |
| $F_{net}$ (N) | 29.4 | 98 | 15.7 | 1.47 | 2.45 | 2.45 M | ? | 2.87 | ? |
| a ($^m/_s{}^2$) | 5.9 | 6.5 | 1.6 | 0.24 | 0.24 | 1.09 | ? | 1.44 | ↘ if cos θ > m/M<br>↖ if cos θ < m/M |

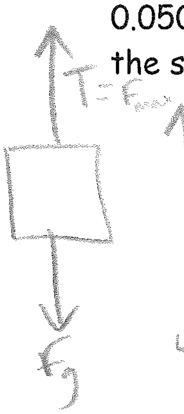
see above (no room here)

see above

## Vector Dynamics Worksheet #5

Key
(name)

1. An elevator, mass 4250 kg, is to be designed so that the maximum acceleration is 0.0500 g. What are the maximum and minimum forces the motor should exert on the supporting cable?

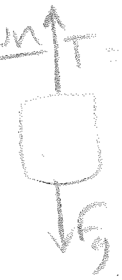
up 

$F_{net} = ma$

$F_{net} = T - F_g$

$4250 \times 9.8 \times 0.05 \times 9.8 = T - 4250\,kg \times 9.8\,N/kg$

$F_{max} = T$ $4.37 \times 10^4\ N$

down 

$F_{net} = ma$

$F_{net} = F_g - T$

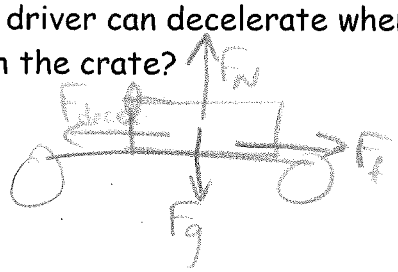
$4250 \times 9.8 \times 0.05 \times 9.8 = 4250 \times 9.8 - T$

$F_{min}$ $T$ $3.96 \times 10^4\ N$

$F_{max}$ = $4.37 \times 10^4\ N$ ✓

$F_{min}$ = $3.96 \times 10^4\ N$

2. A flatbed truck is carrying a 2800 kg crate of machinery. If the coefficient of friction between the crate and the truck bed is 0.55, what is the maximum rate the driver can decelerate when coming to a stop in order to avoid crushing the cab with the crate?



$\mu = 0.55$

$F_f = \mu F_N = \mu\, 2800\,kg \times 9.8\,N/kg = 15092\ N$

$F_{net} = 0 = F_{decel} - F_f$

$F_{decel} = F_f$

$ma = 15092\ N$

$a = \frac{15092\ N}{2800\ kg}$

a = $5.4\ m/s^2$ ✓

3. If the coefficient of friction between a 25 kg crate and the floor is 0.45, how much force is required to move the crate at a steady speed across the floor?

$a = 0$ $F_{net} = 0$

$F_{net} = 0 = F_{app} - F_{fric}$

$F_f = F_{app}$

$\mu F_N =$

$0.45(25\,kg \times 9.8\,N/kg) =$

F = 110 N ✓

4. A force of 270 N is required to start a 40 kg box moving across a concrete floor. What is the coefficient of friction between the box and the floor?

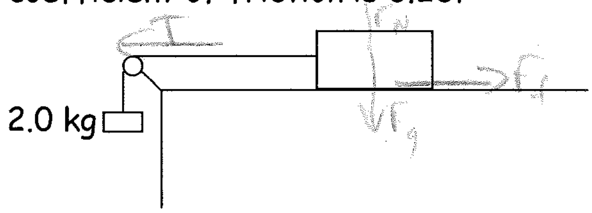
$F_f = F_{app} = 270\,N$ $\qquad \mu = \frac{F_f}{F_N}$

$\mu = \frac{270\,N}{40 \times 9.8\,N}$

$\mu$ = 0.69

5. What mass must the crate have to prevent any motion from occurring if the coefficient of friction is 0.20?

$F_N = F_g$ here




$\mu = \frac{F_f}{F_N} \rightarrow 0.20 = \frac{19.6\,N}{m(9.8\,N/kg)}$

$m = 10\,kg$

$F_{net\,x} = 0 = T - F_f$

$F_f = T = 2\,kg \times 9.8\,N/kg = 19.6\,N$

m = 10 kg

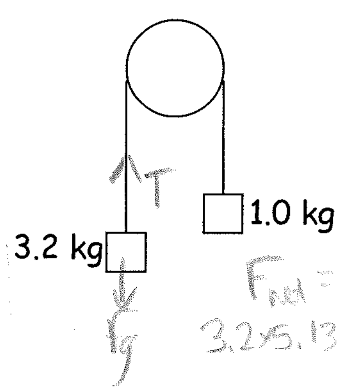
6. Assuming that the pulley in the diagram is massless and frictionless, calculate the acceleration of the 3.2 kg mass and the tension in the cord supporting this mass.

$m_T = 4.2\,kg$

$F_{net} = 3.2\,kg(9.8\,N/kg) - 1.0\,kg \times 9.8\,N/kg$

$m_T a = 21.56$

$a = \frac{21.56}{4.2\,kg} = 5.13\,m/s^2$




$F_{net} = F_g - T$

$3.2 \times 5.13 = 3.2 \times 9.8 - T$

$T = 15\,N$

a = 5.1 m/s²

T = 15 N

Answers: 1. 43 700 N up/39 600 N down

2. 5.4 $^m/_s{}^2$ 3. 110 N 4. 0.69 5. 10 kg 6. 5.1 $^m/_s{}^2$, 15 N

**Vector Dynamics Worksheet #6** Key (name)

1. A falling ball has a mass of 2.0 kg, and the upward force of air resistance is 11.6 N. What is the ball's acceleration?


$\downarrow F_{net} = ma$

$F_{net} = ma = F_g - 11.6N$

$a = \frac{2 \times 9.8N - 11.6N}{2 kg}$

a = 4.0 m/s² ✓

2. A golf ball of mass 60 g is struck by a club and acquires a speed of 80 $^m/_s$ during the impact, which lasts $2.0 \times 10^{-4}$ s. What force is exerted on the ball?

$F = ma$

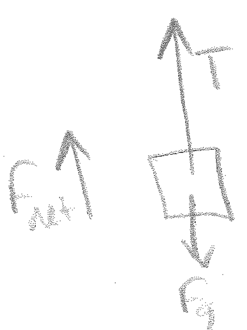
$a = ?$
$V_0 = 0$
$V_f = 80 m/s$
$t = 2 \times 10^{-4} s$

$a = \frac{V_f - V_0}{t} = \frac{(80 - 0) m/s}{2 \times 10^{-4} s} = 4 \times 10^5 m/s^2$

$F = ma = (0.06 kg)(4 \times 10^5 m/s^2)$

F = $2.4 \times 10^4$ N ✓

3. A vertical rope is attached to a 35 kg cart. What tension in the rope is needed to cause the cart to acquire an upward velocity of 4.0 $^m/_s$ in 0.50 s?




$F_{net} = T - F_g = ma$

$T = F_g + ma$

$= 35 \times 9.8 + 35 \times 8 m/s^2$

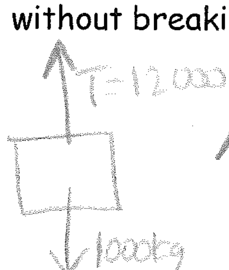
$= 623 N$

$a = ?$
$V_0 = 0$
$V_f = 4 m/s$
$t = .5 s$

$a = \frac{4 - 0}{.5} = 8 m/s^2$

T = $6.2 \times 10^2$ N ✓

4. An elevator of mass 1000 kg is supported by a cable that can sustain a force of 12,000 N. What is the maximum upward acceleration that can be given the elevator without breaking the cable?



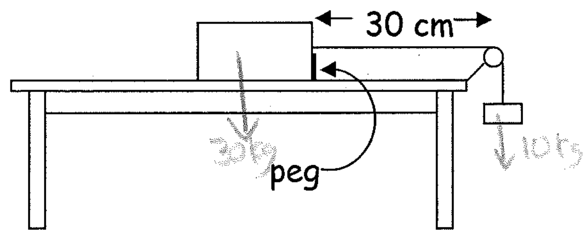

$\uparrow F_{net}$

$F_{net} = m\overset{?}{a} = T - F_g$

$a = \frac{(12000 - 1000 \times 9.8)N}{1000 kg}$

F = 2.2 m/s² ✓

5. In the diagram to the right, the frictional force between the 30 kg block and the table is negligible. The mass of the small block is 10 kg.




(a) If the peg is removed, what is the acceleration?

$F_{net} = 10kg \times 9.8 N/kg$

$ma = 98 N$

$a = \frac{98 N}{40 kg} =$

a = $2.45 m/s^2$.

(b) How long will it take the sliding block to hit the pulley?

$V_0 = 0$

$a = 2.45 m/s^2$

$d = 0.3 m$

$t = ?$

$V_f^2 = V_0^2 + 2ad$

$V_f = \sqrt{2(2.45)0.3}$

$= 1.212 m/s$

$t = \frac{V_f - V_0}{a} = \frac{1.212 - 0}{2.45}$

t = 0.49 s.

(c) What is the tension in the cord while the block is moving?

$F_{net} = ma = F_g - T$

$T = 10 \times 9.8 N - 10 \times 2.45 m/s^2$

$= 73.5 N$

T = 74 N.

(d) What is the tension in the cord after the block ceases to move?

$F_{net} = 0 = F_g - T$

$F_g = T \quad 10 \times 9.8$

T = 98 N.

Answers: 1. 4.0 $m/s^2$ 2. 2.4 x $10^4$ N 3. 620 N 4. 2.2 $m/s^2$
5 a) 2.45 $m/s^2$ b) 0.49 s c) 74 N d) 98 N